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THE ELASTIC CONSTANTS FOR WROUGHT ALUMINUM ALLOYS

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SUMMARY

There are several constants which have been devised as numerical representations of the behavior of metals under the action of loadings which stress the metal within the range of elastic action. Some of these constants, such as Young's modulus of elasticity in tension and compression, shearing modulus of elasticity, and Poisson's ratio, are regularly used in engineering calculations. Precise* tests and experience indicate that these elastic constants are practically unaffected by many of the factors which influence the other mechanical properties of materials and that a few careful determinations under properly controlled conditions are more useful and reliable than many determinations made under less favorable conditions.

It is the purpose of this paper to outline the methods employed by the Aluminum Research Laboratories for the determination of some of these elastic constants, to list the values that have been determined for some of the wrought aluminum alloys, and to indicate the variations in the values that may be expected for some of the commercial products of these alloys.

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*In this discussion, from the viewpoint of the designing engineer's interest, "precise" is intended to mean within limits of error of about 1 percent. The authors appreciate that other more precise methods are available, with which the elastic constants of metals can be determined within closer limits. Such degree of precision, however, is generally of more interest to the physicist and the academic research worker than to the designing engineer.

METHODS OF DETERMINATION OF ELASTIC CONSTANTS

Since the evaluation of an elastic constant of a metal involves the precise determination of the strains resulting from stresses set up in the material by applied loads, it is essential that the loads be known within suitable limits of accuracy. The loads are usually applied by means of one of the many types and sizes of commercially available testing machines. Even though well-recognized standard methods (reference 1) of verification of testing machines are followed in calibrating the machines to be used in determining the elastic constants, yet it should be noted that these standard methods of verifications are primarily intended to check the machines within limits suitable for acceptance testing of materials against product specifications. Present verification standards require that a testing machine shall have errors in its load readings not greater than ±1 percent, within the loading range to be used.

For the precise determination of the elastic constants of metals, smaller limits for the load errors are necessary. This may not require a testing machine with smaller errors but (a) the magnitude of the errors must be known within closer limits; (b) the sensitivity of the machine must be commensurate with the smaller error limits; (c) the reproducibility of the indicated loads must be within about the same smaller limits; and (d) the loads must be indicated within suitable limits under the conditions used during testing (reference 2).

In attempting to meet these requirements the Aluminum Research Laboratories have made three separate calibrations of each load range of the testing machine to be used; the average of these three sets of data compared with the data from the National Bureau of Standards' report on the calibrating device used; and the loads indicated by the testing machine corrected in accordance with the observed errors. Comparisons of the individual readings for each of the different loads used in the three calibration runs show the sensitivity and reproducibility limits obtaining. Very slow rates of loading are used to minimize if not eliminate any dynamic effects.

The calibrating devices used, proving rings, Amsler boxes, and similar elastic devices, have been calibrated by the National Bureau of Standards with their dead-weight machines (reference 3) and found to have errors within less than ±0.2 percent. The errors in the National Bureau of Standards dead-weight machines are less than 0.02 percent and the maximum permissible deviation of each reading of an elastic calibrating device is ±0.2 percent from an average of at least three readings under the same load. Using these calibrating devices and following the procedure outlined it is possible to determine the tensile and compressive loads used in making precise tests within ±0.3 percent.

Similar consideration must be given to the verification of the strainometers to be used in making precise determinations of the elastic constants of metals. Unfortunately such standard methods are not in existence at the present time. Proposed standard methods, however, have been prepared by a member of the staff of the National Bureau of Standards and published by the American Society for Testing Materials (reference 4). In these proposed methods, strainometers are classified into five different groups of which Class A is the one suitable for the precise determination of the elastic constants of metals. The maximum permissible error of indicated strain for this class of strainometers is 0.00001 inch per inch. The Tuckerman and Martens mirror apparatus (reference 5) are representative of the type of strainometer which would come within this proposed Class A. The Aluminum Research Laboratories have a Tuckerman autocollimator and four optical strain gages each of which has been individually calibrated by the National Bureau of Standards (reference 6), and suitable constants furnished for use with each of the gages. These calibration results indicate that these gages will give strain values within about ±0.2 percent.

This Tuckerman apparatus and the National Bureau of Standards' calibration data for it have been used by the Aluminum Research Laboratories to check the Templin strainometer calibrating device (reference 7). This device is used to check other strainometers such as the Martens mirror apparatus, Ewing, Riehle, and Olsen extensometers, Huggenberger tensometers, and so forth. The results obtained in checking the strainometer calibrating device with the Tuckerman apparatus indicated that the calibration factor for the device, on the basis of many check runs varying by about 0.1 percent, could be determined within about ±0.2 percent, corresponding to the error limit mentioned previously for the Tuckerman apparatus.

It is necessary in the precise determination of the elastic constants of metals to load the specimens uniformly and axially so that the stresses induced in the specimens will be as uniform as possible (reference 8). Experience has shown that it is usually necessary to provide special fixtures or devices to insure uniform stressing of the specimens within closer limits than would be obtained with the devices generally provided with commercial testing machines. In the case of tensile specimens satisfactory results have been obtained by the use of threaded adapters, supported in the testing machine by suitably designed, precisely made, and properly lubricated spherically-seated tension bolts. Other types of shackles (references 9 and 10) also have given satisfactory results. Using such equipment it has been found by actual tests that the deviation of strain from the average value, as measured at various locations around the specimen, will be within 1 percent. Averaging strain readings from opposite elements of the specimen will reduce this error close to that of the strainometers used.

For specimens under compression loads, greater difficulties are often encountered in obtaining uniform stressing when using the commercial testing machines available. These difficulties arise from such factors as out-of-parallelism of the testing machine platens, errors in the pitch of the main screws of the testing machine, and elastic distortion of the platens bearing against the ends of the specimen. The use of a suitably designed subpress (reference 11) or platens (reference 12) in the testing machine minimizes or eliminates many of the major troubles encountered in obtaining uniform stressing of the compression specimens. In data obtained with such apparatus the deviations from the average stress should be not more than those indicated for the tensile test (within 1 percent), and as in the case of the tensile test, averaging strains from opposite elements of the specimen will reduce this error close to the errors of the strainometers.

The tensile test specimen ordinarily used for acceptance tests of metals is not particularly well suited for use in determining the elastic constants. A suitable specimen (a) should have a uniform cross section throughout its reduced section; (b) should have the axis of its reduced section coincident with the axis of the ends of the specimen; (c) should have a reduced section length appreciably longer than the gage length; (d) should have a gradual transition in cross section between the reduced section and the ends of the specimen; (e) should be straight throughout its length; (f) should have a scheme of gripping that will insure, insofar as possible, uniform stressing during loading (accurately chased threaded ends or precisely machined shouldered ends will meet this requirement); (g) should be machined smooth and carefully so that any residual stresses or heating effects will be at a minimum and at least negligible; (h) should be from a product free from internal strains that will cause distortion and hence require straightening of the specimen during preparation; and (i) the cross-sectional area of the reduced section and the gage length should be determined within suitable limits, say within 0.1 percent. The type of specimen used by the Aluminum Research Laboratories for determining the tensile and compressive elastic constants is shown in figure 1, and in the photograph (fig. 2).

Here it may be emphasized that the procedure outlined is for the purpose of determining the elastic constants of the metal and not for determining the apparent effects on the constants of many of the factors involved in the testing procedure. The effects of these factors on the values of tensile and yield strength, for example, may be of little practical significance, yet be appreciable in their effects on the elastic constant values.

In all of the calibration and testing procedures just discussed, consideration must be given to the effects of temperature. This is regularly done by the National Bureau of Standards in their checking of load calibrating devices and was done in their calibration of the Tuckerman strain apparatus mentioned. It was also taken into account by the Aluminum Research Laboratories in checking their strainometer calibrating device with the Tuckerman apparatus. The elastic constant values herein given were determined at temperatures close to those obtaining for the calibration work and care was exercised to prevent any appreciable temperature changes during the tests of a given specimen. When determining the elastic constants of a material having a relatively high coefficient of thermal expansion, temperature control is essential for precise results.

The elastic constant stress-strain data, obtained using the testing procedure just described, should be treated by more refined methods than generally used in determining the other mechanical properties. Among the procedures that can be used, the ones suggested by McVetty and Mochel (reference 13) and Tuckerman (reference 14) have been found quite satisfactory and are regularly used by the Aluminum Research Laboratories.

METHODS FOR SHEAR MODULUS

At the present time there are no standard methods for determining the shear modulus of metals. The values given in this paper were obtained from torsion tests of solid round specimens tested in an Amsler torsion machine (Type 150/300 T87) (reference 15) which is provided with a dead-weight calibrating device and a Martens mirror troptometer. Calibration with standard dead weights (50 lb each) checked against a master weight certified by the National Bureau of Standards showed a maximum error for any of the torque ranges used of not more than 0.3 percent. The errors of the troptometer are believed to be less than 0.2 percent.

POISSON'S RATIO

The Poisson's ratio values given have been calculated from the well-known relationship existing between Young's modulus and the shear modulus:

$$\mu = \frac{E - 2E_s}{2E_s} \quad \text{or} \quad \frac{E}{2E_s} - 1$$

The values thus obtained indicate an average value for Poisson's ratio of about 1/3, and this value is recommended as suitable for most engineering design purposes.

VALUES OF ELASTIC CONSTANTS FOR COMMERCIAL WROUGHT ALUMINUM ALLOYS

Values of the modulus of elasticity of wrought aluminum alloys in tension, compression, and shear determined in accordance with methods indicated above, are given in table I. Tests have indicated that a variation of only about 1 or 2 percent is to be expected in the tensile and compressive values in various lots and forms of a given wrought alloy even though the composition, heat treatment, and cold work introduced in fabrication vary throughout the complete range permitted by good commercial practice. The data pertinent to the shear modulus values are somewhat meager but by inference corresponding small differences would be anticipated with variations in the factors just noted. The extreme variations in these constants of 5 to 10 percent or more, found in the literature, are undoubtedly the result of the use of testing methods unsuited for the precise determination of these constants. A testing technique may be entirely satisfactory for the determination of tensile strength, tensile yield strength, and elongation and yet be quite inadequate for the precise determination of modulus of elasticity.

Since the modulus of elasticity is practically constant for a given wrought aluminum alloy product, and since precise values are available, it is rarely necessary for ordinary engineering and research purposes to attempt to determine the modulus for a given lot of such material involved in any project. For most cases it is quite satisfactory to accept the precise values of modulus listed in the accompanying table. It may, however, be desirable to determine the stress-strain curve for individual lots of material because the part of the stress-strain curve above the proportional limit may vary for different lots and for different product forms of the same material (references 16, 17, and 18). Such an individually determined stress-strain curve, while entirely satisfactory for indicating the shape of the curve, may have a slope for the initial straight-line portion different from the nominal modulus of elasticity of the material. The reason for this difference often will be found in the various errors involved in testing, and the magnitude of the difference can usually be considered a measure of the over-all effects of the errors obtaining.

CORRECTION OF DATA FOR ERRORS IN TESTING

Correction of a stress-strain curve for errors in testing can be done by adjusting the slope of the initial straight line to equal the nominal modulus of elasticity and applying a suitable consistent correction to the remainder of the curvo. The nature of the correction applied to the remainder of the curve, of course, will depend upon what type of errors are suspected of being responsible for the faulty initial slope. The following types of errors are encountered:

1. Errors in magnitude of strain, the error being approximately proportional to the load.

Example:

> The approximate strain on a compression member may be measured simply by placing a dial gage arrangement between the platens of the testing machine so that the movement of one head relative to the other is measured. This measurement will include not only the change in length of the specimen but also the elastic distortions of the heads of the testing machine, which in a short specimen may prove to be an appreciable portion of the total movement. It is reasonable to suppose that this distortion of the heads, being elastic, is proportional to the load on the testing machine.

In such instances the best procedure for correcting the stress-strain curve is that indicated in figure 3.

2. Errors in magnitude of strain, the error being approximately proportional to the strain.

Example:

> Faulty magnification ratio in the strainometer.

In such cases the best method for correcting the stress-strain curve is that indicated in figure 4.

3. Errors in magnitude of loads, the errors being approximately proportional to the load.

Example:

> Errors in load-weighing mechanism of testing machine.

In such cases the best method for correcting the stress-strain curve is that indicated in figure 5.

When the type of error leading to the faulty initial slope is not apparent from a consideration of the methods and apparatus used, it is perhaps best to correct the stress-strain curve by some combination of the three methods. For example, the difference between the correct modulus and the apparent modulus can be divided into three equal parts and a third part of the total correction made, in turn, by each of the three methods suggested. The three steps, corresponding to figures 3, 4, and 5, might be performed as follows:

1. Correct each strain reading so that:

$$\text{Corrected strain} = \text{measured strain} - \frac{K_1 f}{E}$$

where $K_1 = \frac{1}{3}\left(\frac{E}{E_a} - 1\right)$.

2. Further correct each of the above corrected strains so that:

$$\text{Final strain} = K_2 \times \text{above corrected strain},$$

where $K_2 = \frac{2E_a + E}{E_a + 2E} = (\text{approx.})\ 1 - K_1$.

3. Correct each stress reading so that:

$$\text{Final stress} = K_3 \times \text{measured stress},$$

where $K_3 = \frac{3E}{E_a + 2E} = (\text{approx.})\ 1 + K_1$.

When these final stresses and final strains are replotted the new modulus line will have the desired correct slope and the entire curve will probably be much nearer its correct position.

The above suggestions for correcting data are not limited to stress-strain curves but are also applicable in principle to other types of data, such as some load-deflection curves, in which the correct initial slope of the curve can be determined from the known nominal modulus of elasticity of the material.

THE MODULUS OF ELASTICITY OF ALCLAD PRODUCTS

The alclad products consist of a central core of high-strength material protected on each side by an integral layer of a different alloy, usually of lower strength, which is sufficiently anodic to the core material to provide electrolytic protection against corrosive attack. In such materials the determination of modulus of elasticity is considerably complicated by the fact that the core and coating may not only have different moduli but may also have widely different proportional limits. When the pieces are uniformly stressed within the elastic range of both the core and coating the effective modulus of the piece is a weighted average of the moduli of the core and coating. This value is often called the primary modulus of elasticity of the material. When the proportional limit of the coating is exceeded, however, the effective modulus of the alclad material, based on the full thickness of the piece, decreases and quickly approaches a value that is represented by the modulus of elasticity of the core material multiplied by the ratio of the core thickness to the total thickness. This value is often called the secondary modulus of elasticity of the material. For engineering purposes an intermediate average value is sometimes used which is smaller than the primary modulus and larger than the secondary modulus.

When a single sheet of alclad is stressed in flexure instead of direct tension or compression the question of effective modulus of elasticity becomes even more complex because the coating, by virtue of its position on the extreme fibers, exerts a greater effect on the over-all behavior of the piece. Tests have indicated that under these circumstances a piece of alclad 24S-T sheet having a core thickness 90 percent of the total thickness will deflect approximately the same as a piece of nonclad 24S-T having an over-all thickness equal to 93 percent of the total. (See reference 19.) This approach to the problem in cases of flexure of single thicknesses seems more logical than trying to arrive at a value of effective modulus of elasticity.

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TABLE I

ELASTIC CONSTANTS FOR VARIOUS WROUGHT ALUMINUM ALLOYS
DERIVED FROM PRECISE STRESS-STRAIN TESTS

Alloy	Modulus of elasticity, psi x 10^{-6}				Poisson's ratio
	Tension	Compression	Recommended average tension and compression	Shear	
2S	9.9	10.0	10.0	3.8	0.32
53S	9.9	10.1	10.0	3.8	.32
61S	9.9	10.1	10.0	3.8	.32
52S	10.1	10.2	10.2	3.85	.33
A51S	10.1	10.3	10.2	3.85	.33
75S	10.3	10.5	10.4	3.9	.33
25S	10.3	10.5	10.4	3.9	.33
17S	10.4	10.6	10.5	3.95	.33
14S	10.5	10.7	10.6	4.0	.32
24S	10.5	10.7	10.6	4.0	.32

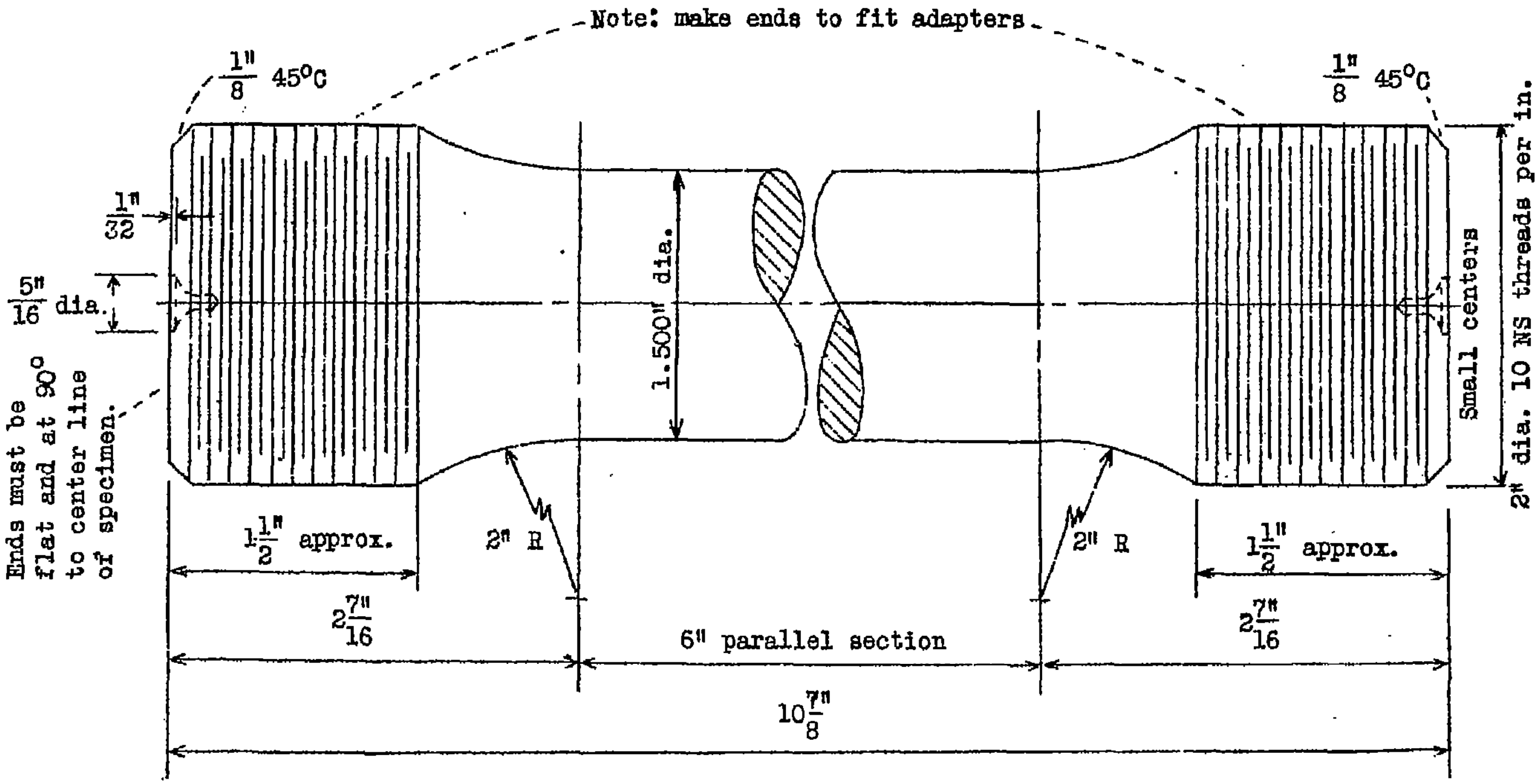


Figure 1.- Specimen for use in determining modulus of elasticity in tension and compression.

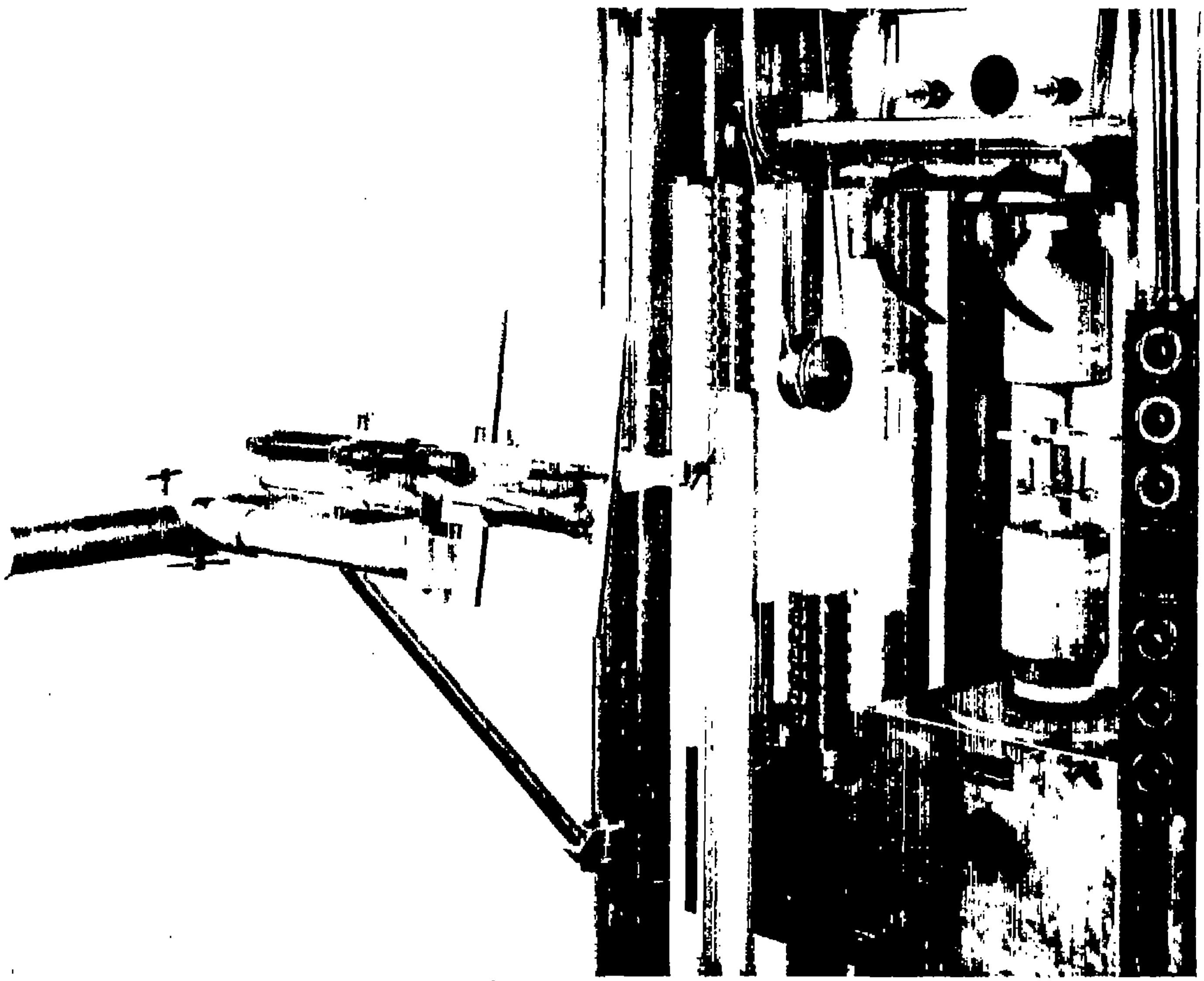


Figure 2.- Martens mirror apparatus and special tension grips as used in determining modulus of elasticity of aluminum alloys.

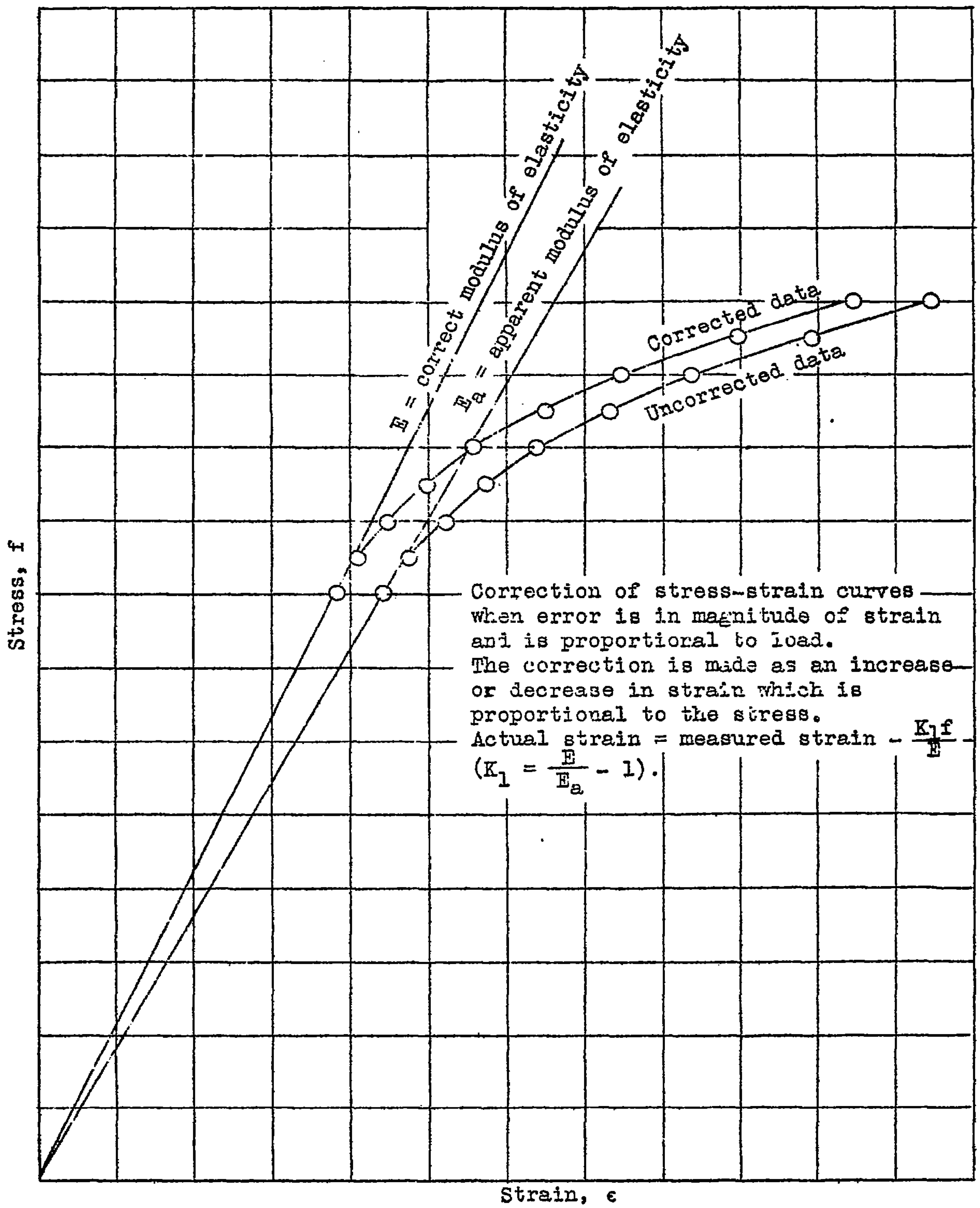
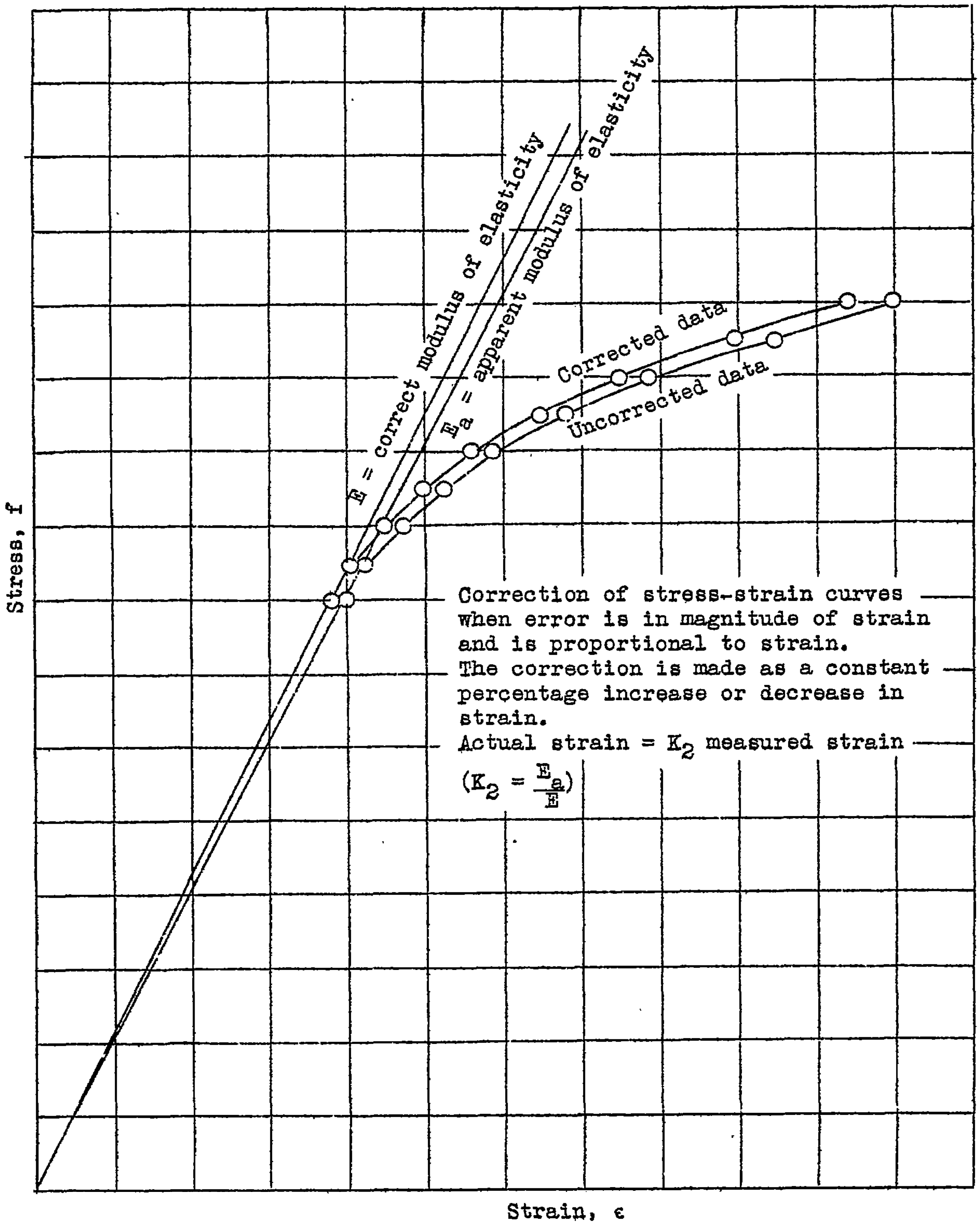

E = correct modulus of elasticity
E_a = apparent modulus of elasticity
Corrected data
Uncorrected data
Correction of stress-strain curves when error is in magnitude of strain and is proportional to load.
The correction is made as an increase or decrease in strain which is proportional to the stress.
Actual strain = measured strain − $\frac{K_1 f}{E}$
$(K_1 = \frac{E}{E_a} - 1)$.
Stress, f
Strain, ε

Figure 3.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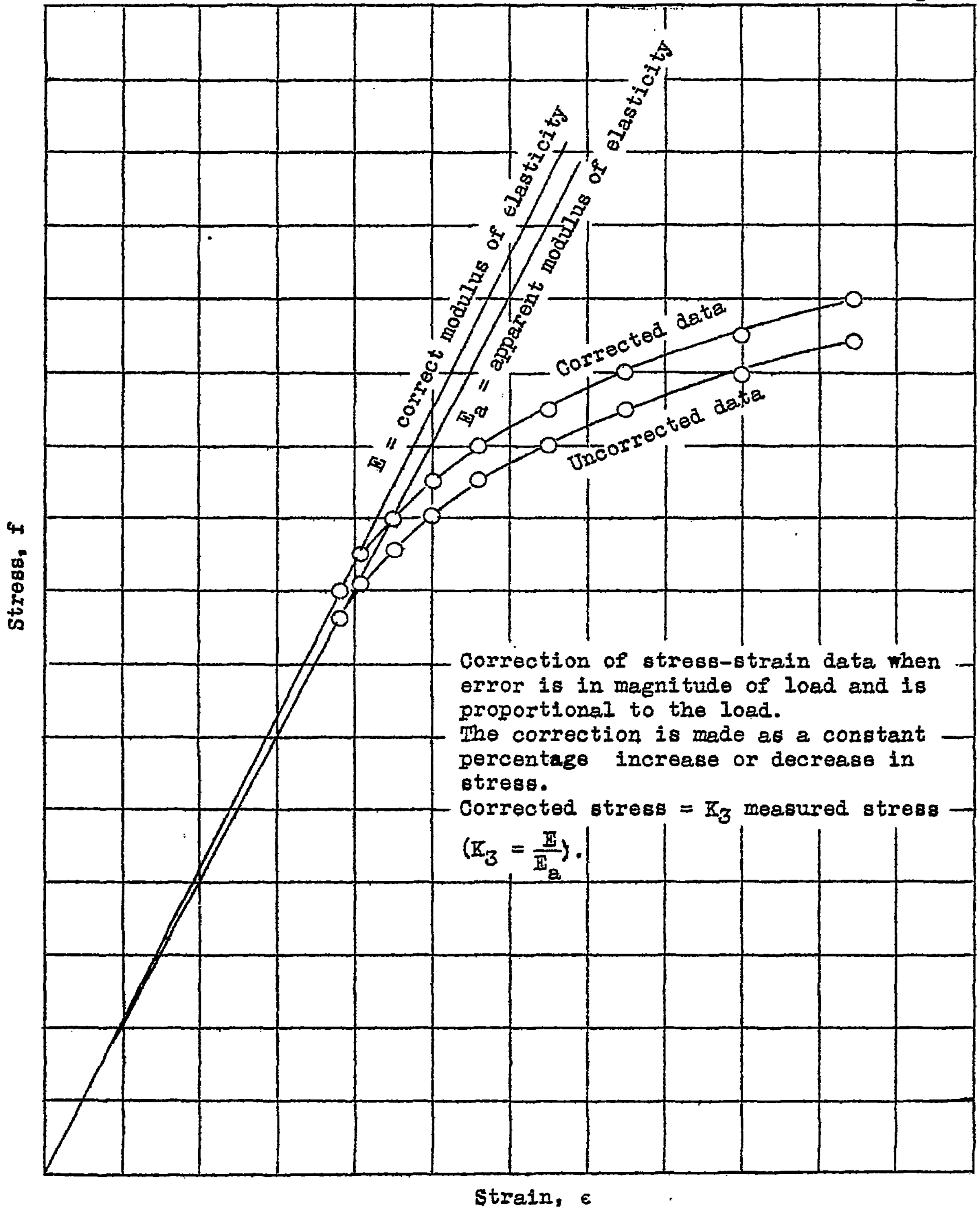
Correction of stress-strain curves when error is in magnitude of strain and is proportional to strain.
The correction is made as a constant percentage increase or decrease in strain.
Actual strain = K_2 measured strain
$(K_2 = \frac{E_a}{E})$
Stress, f
Strain, ε

Figure 4.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Strain, ε
Correction of stress-strain data when error is in magnitude of load and is proportional to the load.
The correction is made as a constant percentage increase or decrease in stress.
Corrected stress = K_3 measured stress
$(K_3 = \frac{E}{E_a})$.

Figure 5.